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Optimal Reset Number of a Microprocessor System with Network Processing

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Abstract—As computer network technology has remarkably developed, microcomputers which form data terminal equipment (DTE) in a communication network have been used in many practical fields and the demand for improvement of their reliabilities has greatly increased. In fact, a microprocessor (μP) which is one of vital devices of a communication network often fails through some faults due to noise and changes in the environment and programming bugs. Therefore, it is necessary to take preventive measures for occurrences of such errors. This paper considers the maintenance problem for improving the reliability of a μP system with network processing. After the system has made a stand-alone processing, it executes successively communication procedures of a network processing. When either μP failures or application software errors in the system have occurred, a μP is reset to the beginning of its initial state and restarts again. The reliability quantities such as the mean time to the success of a network processing and the expected reset number, using the theory of Markov renewal processes, are derived. An optimal reset number, which minimizes the expected cost until a network processing is successful, is analytically discussed. A numerical example is finally given. © 2003 Elsevier Ltd. All rights reserved.

Keywords—Microprocessor, Network processing, Mean time, Expected cost, Reset number.

1. INTRODUCTION

As computer network technology has remarkably developed, microcomputers which form data terminal equipment (DTE) in a communication network have been used in many practical fields. Recently, a new communication network combining the information processing and communication plays an important role as the infrastructure in the information society. Therefore, the demand for improvement of reliabilities and functions for devices of a communication network have greatly increased [1].

In fact, a microprocessor (μP) which is one of vital devices of a communication network often fails through some faults due to noise and changes in the environment and programming bugs. Hence, it is necessary to take preventive measures for occurrences of such errors. Generally, when we consider the reliability of the system on an operational stage, we should regard the cause of error occurrences of a μP as faults of software, such as mistakes of operational control and

memory access, rather than faults of hardware. That is, when errors of a μP have occurred, it is effective to recover the system by the operation of reset [2].

This paper considers the maintenance problem for improving the reliability of a μP system with network processing. We formulate the model of a μP system with network processing, in order to more accurately express the processing operation, by considering the process from the beginning of an initial processing to the success of a network processing: After the system has made a stand-alone processing, it executes successively communication procedures of a network processing. When either μP failures or application software errors in the system have occurred, a μP is reset to the beginning of its initial state and restarts again. Most reliability evaluation models of a μP system until now have assumed that both errors of a μP and failures of the data transmission occur unlimitedly [3–6]. This paper assumes that if the reset due to errors has occurred N times intermittently, then a μP interrupts its processing and restarts again from the beginning of its initial state after a constant time. That is, if the reset has occurred frequently, the system has latent faults, and takes the preventive maintenance to check the environment and to eliminate errors, and then, the system is renewed by its preventive maintenance.

We derive the reliability quantities such as the mean time and the expected reset number until a network processing is successful. Further, we regard the losses and times for the reset and the interruption of processing and for the maintenance to restart the system as expected costs, and discuss optimal policies which minimize them. A numerical example is finally given.

2. MODEL AND ANALYSIS

We pay attention to only a certain DTE which consists of a workstation or a personal computer and connects with some networks, and consider the problem for improving its reliability.

Suppose that errors of a μP system occur according to an exponential distribution $F(t)$ with mean $1/\lambda$. If errors of a μP have occurred, a μP is reset to the beginning of its initial state and restarts again. It is assumed that any reset times are neglected.

- (1) After a μP begins to operate, it executes an initial processing immediately and a stand-alone processing.
- (2) The times for an initial processing and a stand-alone processing have a general distribution $V(t)$ with mean $1/v$ and an exponential distribution $A(t)$ with $1/\alpha$, respectively.
- (3) After a μP completes a stand-alone processing, it begins to execute a network connection processing.
 - (a) A connection processing needs the time according to a general distribution $B(t)$ with mean $1/\beta$, and fails with probability γ ($0 \leq \gamma < 1$).
 - (b) If a connection processing has failed, a μP executes the same processing again after a constant time w where $W(t) \equiv 0$ for $t < w$ and 1 for $t \geq w$.
- (4) After a connection processing has been successful, a μP executes a network processing.
 - (c) A network processing needs the time according to a general distribution $U(t)$ with mean $1/u$, and is successful with probability 1 if it has not failed.
- (5) If the N^{th} reset has occurred since a μP begins to operate, once it interrupts the processing, it restarts again from the beginning after a constant time μ , where $G(t) \equiv 0$ for $t < \mu$ and 1 for $t \geq \mu$.

Under the above assumptions, we define the following states of the system.

State 0: An initial processing begins.

State 1: A stand-alone processing begins.

State 2: A stand-alone processing is completed and a network connection processing begins.

State 3: A network connection processing succeeds and a network processing begins.

State F : A network processing is interrupted.

State S : A network processing succeeds.

The system states defined above form a Markov renewal process [7] where State S is an absorbing state.

Let $Q_{i,j}(t)$ ($i = 0, 1, 2, 3$; $j = 0, 1, 2, 3, S$) be one-step transition probabilities of a Markov renewal process. Then, mass functions $Q_{i,j}(t)$ from State i at time 0 to State j at time t are

$$Q_{0,0}(t) = \int_0^t \overline{V(t)} dF(t), \quad (1)$$

$$Q_{0,1}(t) = \int_0^t \overline{F(t)} dV(t), \quad (2)$$

$$Q_{1,0}(t) = \int_0^t \overline{A(t)} dF(t), \quad (3)$$

$$Q_{1,2}(t) = \int_0^t \overline{F(t)} dA(t), \quad (4)$$

$$Q_{2,0}(t) = \sum_{j=1}^{\infty} X^{(j-1)}(t) * \int_0^t [\overline{B(t)} + \gamma B(t) * \overline{W(t)}] dF(t), \quad (5)$$

$$Q_{2,3}(t) = \sum_{j=1}^{\infty} X^{(j-1)}(t) * \left[(1 - \gamma) \int_0^t \overline{F(t)} dB(t) \right], \quad (6)$$

$$Q_{3,0}(t) = \int_0^t \overline{U(t)} dF(t), \quad (7)$$

$$Q_{3,S}(t) = \int_0^t \overline{F(t)} dU(t), \quad (8)$$

where

$$X(t) \equiv \gamma \int_0^t \overline{F(t)} dB(t) * \int_0^t \overline{F(t)} dW(t), \quad (9)$$

the asterisk mark denotes the Stieltjes convolution, and $a^{(n)}(t)$ denotes the n -fold Stieltjes convolution of a distribution $a(t)$ with itself; i.e., $a^{(n)}(t) \equiv a^{(n-1)}(t) * a(t)$, $a(t) * b(t) \equiv \int_0^t b(t-u) da(u)$.

We derive the mean time ℓ_S from the beginning of system operation until a network processing is successful. Let $H_{0,S}(t)$ be the first-passage time distribution from State 0 to State S . Then we have

$$H_{0,S}(t) = \sum_{j=1}^N D^{(j-1)}(t) * Z(t), \quad (10)$$

where

$$D(t) \equiv Q_{0,0}(t) + Q_{0,1}(t) * Q_{1,0}(t) + Q_{0,1}(t) * Q_{1,2}(t) * Q_{2,0}(t) \\ + Q_{0,1}(t) * Q_{1,2}(t) * Q_{2,3}(t) * Q_{3,0}(t), \quad (11)$$

$$Z(t) \equiv Q_{0,1}(t) * Q_{1,2}(t) * Q_{2,3}(t) * Q_{3,S}(t). \quad (12)$$

It is noted that $D(t)$ is the distribution function in which a μP is reset by occurrences of errors and $Z(t)$ is the distribution function which the system moves from State 0 to State F directly without being reset. Further, the first-passage time distribution $H_{0,F}(t)$ from State 0 to State F by a μP the N^{th} reset is given by

$$H_{0,F}(t) \equiv D^{(N)}(t). \quad (13)$$

Therefore, the first-passage time distribution $L_S(t)$ until a network processing is successful is given by the following renewal equation:

$$L_S(t) = H_{0,S}(t) + H_{0,F}(t) * G(t) * L_S(t). \quad (14)$$

Let $\phi(s)$ be the Laplace-Stieltjes (LS) transform of any function $\Phi(t)$; i.e., $\phi(s) \equiv \int_0^\infty e^{-st} d\Phi(t)$. Taking the LS transforms on both sides of (14) and arranging them, we have

$$l_S(s) = \frac{h_{0,S}(s)}{1 - h_{0,F}(s)g(s)}. \quad (15)$$

Hence, the mean time ℓ_S is given by

$$\ell_S \equiv \int_0^\infty t dL_S(t) = \lim_{s \rightarrow 0} \left\{ -\frac{dl_S(s)}{ds} \right\} = -\frac{z'(0) + d'(0)}{1 - d(0)} + \frac{\mu d(0)^N}{1 - d(0)^N}, \quad (16)$$

where $\phi'(s)$ is the differential function of $\phi(s)$; i.e., $\phi'(s) \equiv \frac{d\phi(s)}{ds}$. From equation (16), ℓ_S is strictly decreasing in N and is minimized when $N = \infty$.

Next, we derive the expected reset number M_R from the start of system operation or the restart by the reset until a network processing is successful. Let $M_R(t)$ be the expected reset number until a network processing is successful in an interval $(0, t]$. Then, we have

$$M_R(t) = \sum_{j=1}^{N-1} j D^{(j)}(t) * Z(t). \quad (17)$$

Thus, the expected reset number is given by

$$M_R \equiv \lim_{t \rightarrow \infty} M_R(t) = \lim_{s \rightarrow 0} \sum_{j=1}^{N-1} j [d(s)]^j z(s) = \frac{d(0)}{1 - d(0)} [1 - N d(0)^{N-1} + (N-1) d(0)^N], \quad (18)$$

where it is noted that $z(0) = 1 - d(0)$.

Further, let $M_F(t)$ be the distribution of the expected interruption number of processing from the start of system operation until a network processing is successful. Then, we have the following renewal equation:

$$M_F(t) = H_{0,F}(t) * [1 + G(t) * M_F(t)]. \quad (19)$$

Similar to equation (18), the expected interruption number M_F until a network processing is successful is given by

$$M_F = \frac{d(0)^N}{1 - d(0)^N}. \quad (20)$$

3. OPTIMAL POLICIES

We obtain two objective functions which are the total expected cost $C(N)$ and the expected cost $\hat{C}(N)$ per unit of time until a network processing is successful, and discuss optimal policies which minimize them, respectively.

3.1. Policy 1

Let c_1 be the cost for the reset and c_2 be the cost for an interruption of processing. Then, we define the total expected cost $C(N)$ until a network processing is successful as the following equation:

$$C(N) \equiv c_1 M_R + c_2 M_F = c_1 \left[\frac{D(1 - D^N)}{1 - D} - N D^N \right] + \frac{c_2 D^N}{1 - D^N}, \quad N = 1, 2, \dots, \quad (21)$$

where $D \equiv d(0)$ which is the probability that a μP is reset.

We seek an optimal number N^* which minimizes $C(N)$. From the inequality $C(N+1) - C(N) \geq 0$, we have

$$N(1-D^N)(1-D^{N+1}) \geq \frac{c_2}{c_1}. \quad (22)$$

Denoting the left-hand side of (22) by $L(N)$, we have

$$L(1) = (1-D)(1-D^2), \quad (23)$$

$$L(\infty) = \infty. \quad (24)$$

Hence, $L(N)$ is strictly increasing in N from $L(1)$ to ∞ . Thus, we have the following optimal policy.

- (i) If $L(1) < c_2/c_1$, then there exists a finite and unique minimum $N^*(> 1)$ which satisfies (22).
- (ii) If $L(1) \geq c_2/c_1$, then $N^* = 1$ and the total expected cost is $C(1) = (c_2 D)/(1-D)$.

In this model, c_1 is the cost for the increase of system resources such as spaces of memory and times by the reset, and c_2 is for the increase of system resources by the preventive maintenance to eliminate the cause of errors. It could be generally estimated that c_2 is greater than c_1 ; i.e., $c_2 \geq c_1$. Thus, we have $L(1) < c_2/c_1$, and hence, $N^* > 1$. Further, it is easily shown that N^* increases with c_2/c_1 .

3.2. Policy 2

In Policy 1, we have considered the total expected cost as an objective function. However, it would be more practical to introduce the measure of the time until a network processing is successful. Next, we consider an optimal policy which minimizes the expected cost per unit of time until a network processing is successful. That is, from equations (16) and (21), we define the expected cost $\hat{C}(N)$ per unit of time as the following equation:

$$\hat{C}(N) \equiv \frac{C(N)}{\ell_S} = \frac{c_1 \sum_{j=1}^{N-1} j D^j (1-D) - (A/\mu) c_2}{A + \mu D^N / (1-D^N)} + \frac{c_2}{\mu}, \quad N = 1, 2, \dots, \quad (25)$$

where

$$A \equiv -\frac{z'(0) + d'(0)}{1-D} > 0. \quad (26)$$

We seek an optimal number N_1^* which minimizes $\hat{C}(N)$. From the inequality $\hat{C}(N+1) - \hat{C}(N) \geq 0$, we have

$$N(1-D^N)(1-D^{N+1}) + \frac{\mu}{A} \left[N D^N (1-D^{N+1}) + (1-D) \sum_{j=1}^{N-1} j D^j \right] \geq \frac{c_2}{c_1}. \quad (27)$$

Denoting the left-hand side of (27) by $L_1(N)$,

$$L_1(1) = (1-D^2) \left(1-D + \frac{\mu}{A} D \right), \quad (28)$$

$$L_1(\infty) = \infty. \quad (29)$$

Putting the second term on the bracket of the left-hand side of (27) by

$$L_2(N) \equiv N D^N (1-D^{N+1}) + (1-D) \sum_{j=1}^{N-1} j D^j, \quad (30)$$

we have

$$L_2(1) = (1 - D^2) D, \quad (31)$$

$$L_2(\infty) = \frac{D}{1 - D}, \quad (32)$$

$$L_2(N + 1) - L_2(N) = D^{N+1} [1 - D^{N+2} + ND^N (1 - D^2)] > 0. \quad (33)$$

Hence, $L_2(N)$ is strictly increasing in N . Further, since $N(1 - D^N)(1 - D^{N+1})$ in (27) is also strictly increasing in N , $L_1(N)$ is strictly increasing in N from $L_1(1)$ to ∞ . Thus, we have the following optimal policy.

- (i) If $L_1(1) < c_2/c_1$, then there exists a finite and unique minimum $N_1^* (> 1)$ which satisfies (27).
- (ii) If $L_1(1) \geq c_2/c_1$, then $N_1^* = 1$, and the resulting cost is

$$\hat{C}(1) = \frac{c_2 D}{A(1 - D) + \mu D}. \quad (34)$$

Further, we compare the optimal Policy 2 to the optimal Policy 1. Since from equations (22) and (27),

$$L_1(N) - L(N) = \frac{\mu}{A} \left[ND^N (1 - D^{N+1}) + (1 - D) \sum_{j=1}^{N-1} j D^j \right] > 0, \quad N = 1, 2, \dots, \quad (35)$$

hence, $N^* \geq N_1^*$.

This means that when the number N of reset is small, the mean time until a network processing is large, since ℓ_S strictly decreases in N . Thus, it would be better to adopt Policy 2 where N is small when we consider only the cost of the system on the whole. On the other hand, if we want a processing time to be small, we should adopt Policy 1.

4. NUMERICAL EXAMPLE

We compute numerically the optimal number N_1^* which minimizes $\hat{C}(N)$ for Policy 2. Suppose that the mean initial processing time $1/v$ of μP is a unit of time and the mean time to error occurrences is $(1/\lambda)/(1/v) = 30\text{--}60$. Further, the mean stand-alone processing time is $(1/\alpha)/(1/v) = 5\text{--}20$, the mean network connection processing time is $(1/\beta)/(1/v) = 1$, the mean waiting time when a network connection processing fails is $w/(1/v) = 1\text{--}4$, the mean network processing time is $(1/u)/(1/v) = 10$, the mean maintenance time after an interruption of processing is $(1/\mu)/(1/v) = 10$, the probability that a network connection processing fails is $\gamma = 0.2, 0.4, 0.6$, and the cost c_1 for the reset is a unit of cost and the cost rate of an interruption of processing is $c_2/c_1 = 1\text{--}3$.

Table 1 gives the optimal reset number N_1^* which minimizes the expected cost $\hat{C}(N)$. For example, when $(1/\lambda)/(1/v) = 60$, $wv = 2$, $\gamma = 0.2$, $(1/\alpha)/(1/v) = 10$, and $c_2/c_1 = 2$, the optimal number is $N_1^* = 3$.

This shows that the optimal number N_1^* decreases with $(1/\lambda)/(1/v)$, however, increases with wv , γ , $(1/\alpha)/(1/v)$, and c_2/c_1 . This can be interpreted that when the cost for an interruption of processing is large, N_1^* increases with c_2/c_1 , and so, the processing should not be excessively interrupted. That is, we should keep on executing the processing as long as possible by the reset. Table 1 also shows that N_1^* depends on each parameter when $(1/\lambda)/(1/v)$ is small, i.e., when errors of a μP occur frequently; however, N_1^* depends little on wv , γ , and $(1/\alpha)/(1/v)$ when $(1/\lambda)/(1/v) \geq 60$, and N_1^* is almost determined by c_2/c_1 .

Table 1. Optimal reset number N_1^* to minimize $\hat{C}(N)$.

$(1/\lambda)/(1/v)$	wv	γ	$(1/\alpha)/(1/v) = 5$					$(1/\alpha)/(1/v) = 10$					$(1/\alpha)/(1/v) = 20$				
			c_2/c_1					c_2/c_1					c_2/c_1				
			1	1.5	2	2.5	3	1	1.5	2	2.5	3	1	1.5	2	2.5	3
30	1	0.2	2	2	3	3	4	2	3	3	3	4	2	3	3	4	4
		0.4	2	2	3	3	4	2	3	3	4	4	2	3	3	4	4
		0.6	2	3	3	3	4	2	3	3	4	4	3	3	4	4	4
	2	0.2	2	2	3	3	4	2	3	3	3	4	2	3	3	4	4
		0.4	2	3	3	3	4	2	3	3	4	4	3	3	3	4	4
		0.6	2	3	3	3	4	2	3	3	4	4	3	3	4	4	4
	4	0.2	2	2	3	3	4	2	3	3	4	4	2	3	3	4	4
		0.4	2	3	3	3	4	2	3	3	4	4	3	3	4	4	4
		0.6	2	3	3	4	4	2	3	3	4	4	3	3	4	4	5
60	1	0.2	2	2	3	3	4	2	2	3	3	4	2	2	3	3	4
		0.4	2	2	3	3	4	2	2	3	3	4	2	2	3	3	4
		0.6	2	2	3	3	4	2	2	3	3	4	2	2	3	3	4
	2	0.2	2	2	3	3	4	2	2	3	3	4	2	2	3	3	4
		0.4	2	2	3	3	4	2	2	3	3	4	2	2	3	3	4
		0.6	2	2	3	3	4	2	2	3	3	4	2	2	3	3	4
	4	0.2	2	2	3	3	4	2	2	3	3	4	2	2	3	3	4
		0.4	2	2	3	3	4	2	2	3	3	4	2	2	3	3	4
		0.6	2	2	3	3	4	2	2	3	3	4	2	3	3	3	4

5. CONCLUSIONS

We have investigated the problem for improving the reliability of a μP system with network processing, and have derived the mean time and mean reset numbers until a network processing is successful. Further, we have discussed the optimal reset numbers which minimize the total expected cost and the expected cost per unit of time.

It has been shown from the mathematical analysis that the optimal reset number which minimizes the total cost is larger than that which minimizes the expected cost per unit of time. It has also been shown from the numerical example that the optimal reset number which minimizes the expected cost decreases with the mean time to error occurrences of a μP , however, increases with the mean stand-alone processing time, the probability that a network processing fails, and the cost for an interruption of processing. Further, when the mean time to error occurrences is large, the optimal reset number depends little on each parameter and is almost determined by the cost for an interruption of processing.

It would be very important to evaluate the reliability of a μP system with network processing. Further studies for such subjects would be expected.

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